ON THE EXPANSION INTO A VACUUM OF A RAREFIED PLASMA WITH TWO SORTS OF IONS

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The one-dimensional expansion of a plasma with different temperatures and two sorts of ions into a vacuum is examined. When the ion velocity distribution in the plasma is Maxwellian, propagation of a rarefaction wave is observed, the boundary of which is a weak discontinuity moving with the velocity of ionic sound in the plasma. The value of this velocity is found for the plasma in question. Attention is mainly focused on finding the first two moments of the distribution functions, i.e., the mean velocities and the densities of the heavy particles. An approximate asymptotic solution is obtained for the system of transport equations in the case when the two kinds of ions have similar masses, and the system is solved numerically by computer. Some features of the solutions, typifying a plasma in which the different sorts of ions have different masses, are analyzed in detail.

1. Let the plasma occupy the half-space x < 0 at the initial instant and start to expand into a vacuum (x > 0) at the instant t = 0. The plasma is described by the collision-free kinetic equations with self-consistent field

$$\frac{\partial f_e}{\partial t} + v\frac{\partial f_e}{\partial x} + \frac{e}{m}\frac{\partial \varphi}{\partial x}\frac{\partial f_e}{\partial v} = 0 \tag{1.1}$$

$$\frac{\partial f_i}{\partial t} + v\frac{\partial f_i}{\partial x} - \frac{e}{M_i}\frac{\partial f}{\partial x}\frac{\partial f_i}{\partial v} = 0 \qquad (i = 1, 2) \tag{1.2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = -4\pi e(N_1 + N_2 - N_e), \quad N_k = \int_{-\infty}^{\infty} f_k(x, t, v)\,dv \tag{1.3}$$

Here f_k is the distribution of particles of the k-th sort, N_k is the concentration of these particles, φ is the electric field potential, M_i is the mass of an ion of the i-th sort, and m is the mass of an electron.

The problem of expansion into a vacuum was considered in [1] for a simple plasma (with electrons and ions of one sort); this solution will be assumed to hold when the two sorts of ions have equal masses.

The authors of [1] concluded, from a study of the initial stage of the expansion process, that the plasma movement rapidly approximates to a similarity type. For, after quite a short time $t > (M_2/2T_i)^{1/2} \times D_i$, the characteristic dimension of the inhomogeneities in the plasma becomes much greater than the Debye ionic radius D_i. Hence the plasma can be regarded as quasi-neutral from this instant (see [4]), while the electrons have a Boltzmann distribution, which means in the last analysis that the solution of Eqs. (1.2) can be sought in the class of similarity solutions, i.e., we can put $f_i = f_i(x/t, v)$. Introducing the dimensionless variables

$$\tau = \left(\frac{M_1}{2T_e}\right)^{1/2}\frac{x}{t}, \qquad u_i = \left(\frac{M_1}{2T_e}\right)^{1/2} v_i, \qquad g_i = \left(\frac{2\pi T_e}{M_1}\right)^{1/2} f_i$$

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the equations

$$(u_1 - \tau)\frac{\partial g_2}{\partial \tau} - \frac{1}{2}\frac{\partial g_2}{\partial u_1}\frac{d}{d\tau}\left(\ln \int_{-\infty}^{\infty} [g_1 + g_2]\, du\right) = 0$$

$$(u_2 - \tau)\frac{\partial g_2}{\partial \tau} - \frac{\gamma}{2}\frac{\partial g_2}{\partial u_2}\frac{d}{d\tau}\left(\ln \int_{-\infty}^{\infty} [g_1 + g_2]\, du\right) = 0 \qquad \left(\gamma = \frac{M_1}{M_2}\right) \tag{1.4}$$

are obtained for the dimensionless distribution function.

Consider the equations for the first moments of the distribution function, i.e., the mean (directed) velocity v_i and the ion density N_i:

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial x}(N_i v_i) = 0$$

$$M_i\left[\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x}\right] = -T_e \frac{\partial \ln (N_1 + N_2)}{\partial x}\,. \tag{1.5}$$

In dimensionless variables those become

$$(u_1 - \tau)\frac{d\ln N_1}{d\tau} + \frac{du_1}{d\tau} = 0, \qquad (u_2 - \tau)\frac{d\ln N_2}{d\tau} + \frac{du_2}{d\tau} = 0$$

$$(u_1 - \tau)\frac{du_1}{d\tau} + \frac{1}{2}\frac{d\ln (N_1 + N_2)}{d\tau} = 0, \qquad (u_2 - \tau)\frac{du_2}{d\tau} + \frac{\gamma}{2}\frac{d\ln (N_1 + N_2)}{d\tau} = 0\,. \tag{1.6}$$

Taking a cold plasma, i.e., one in which thermal movement of the ions is negligible, Eqs. (1.6) will be assumed to hold for all τ greater than τ_0, to be defined below. The boundary conditions for Eqs. (1.6) may be stated as follows. It is well known (see, e.g., [3]) that the one-dimensional similarity movement of a compressible gas can be represented as propagation of a nonstationary rarefaction wave, the boundaries of which are weak discontinuities, moving with the local velocity of sound. This representation also applies to the present problem, since the physical picture of the process and Eqs. (1.5) are similar to the gas-dynamic case. It will be assumed that the nonstationary rarefaction wave travels in the direction $\tau < 0$, its forward front being a weak discontinuity; to the left of it there is a region of undisturbed plasma with $N_i = n_{i0}$, $u_i = 0$, and to the right a region in which the density and velocity of the ions are described by solutions of the system (1.6). The velocity of the weak discontinuity will first be found for a plasma consisting of ions of one sort. We transform to a coordinate system connected with the discontinuity: x is replaced by the coordinate $\xi = x + at$, where a is the required velocity of propagation of the weak discontinuity, for which the expression

$$a^2 = T_e / M \tag{1.7}$$

is easily obtained.

As was to be expected, this is the same as the velocity of ionic sound in the plasma [4]. On the discontinuity, therefore, i.e., with $\xi = 0$ or $x/t = -a$, the following conditions are satisfied for the solutions $N/N_0 = C \exp(-\tau\sqrt{2})$ and $u = \tau + {}^1/_2\sqrt{2}$ (see [1]):

$$1 = C \exp\left[-\left(\frac{M}{T_e}\right)^{1/2}\frac{x}{t}\right], \qquad 0 = \left(\frac{M}{2T_e}\right)^{1/2}\frac{x}{t} + \frac{1}{\sqrt{2}}\,.$$

Hence we obtain C = 1/e, in agreement with [1]. In our present problem, the plasma consists of two sorts of ions, and the velocities of the discontinuities for these "ionic gases" may in general be distinct. It can be shown, however, that when a rarefaction wave moves through a mixture of fixed gases there is only one weak discontinuity moving in a given direction. This was proved in [5,6] for the ordinary equations of gas dynamics. Consider the system of equations (1.5), on the assumption (see [7]) that all the hydrodynamic functions Φ_i are continuous on the discontinuity, while their derivatives have nonzero jumps equal to

$$[\partial \Phi_i / \partial x] = \lambda_{\Phi_i} \boldsymbol{n}, \qquad [\partial \Phi_i / \partial t] = -\lambda_{\Phi_i} a$$

Here **n** is the unit normal to the surface of the discontinuity, and a is the velocity of the discontinuity. We then have

$$-\lambda_{N_i}a + \lambda_{N_i}v_i n + \lambda_{v_i}N_i n = 0$$
$$a_i(\lambda_{N_1} + \lambda_{N_2}) = -(N_1 + N_2)(\lambda_{v_i}v_i n - \lambda_{v_i}a)$$

where a_i is the velocity of sound in a plasma with ions of the i-th sort. The condition for this system of four equations to be solvable for λ_{N_i} and λ_{V_i} is

$$(a - V_{1n})^2 N_2 a_2^2 + (a - V_{2n})^2 N_1 a_1^2 = (a - V_{1n})^2 (a - V_{2n})^2 (N_1 + N_2)$$

In general, this equation has three nonzero roots, two of which are negative; but when the movement takes place in a fixed mixture ($V_{1n} = V_{2n} = 0$, $N_{1n} = n_{10}$, $N_{2n} = n_{20}$), we have

$$a^2 = \frac{n_{10}a_1^2 + n_{20}a_2^2}{n_{10} + n_{20}}$$

Hence, for the wave traveling in the direction $x < 0$,

$$a = \left(\frac{T_e}{M_1}\frac{\gamma n_{20} + n_{10}}{n_{20} + n_{10}}\right)^{1/2}$$

2. Since the system (1.6) cannot be solved exactly, in addition to solving it numerically we examined its asymptotic properties as $\gamma \to 1$, i.e., when the ions of the two kinds have nearly equal masses (the limiting case $M_1 = M_2$ is covered by [1]). The solution of system (1.5) was then sought in the form

$$u_i = u_0 + \delta v_i, \quad N_i^\circ = N_{oi}^\circ + \delta n_i \quad (N_i^\circ = N_i / N_0) \tag{2.1}$$

Here δv_i and δn_i are small terms added to the "basic" solutions u_0 and N_{0i}°; these latter give respectively the mean velocity and density, i.e., the exact solution of system (1.6) in the limiting case $\gamma = 1$; they were obtained in [2]:

$$\begin{aligned} N_{10}^\circ &= \frac{n_{10}\exp(-\sqrt{2}\tau - 1)}{n_{10} + n_{20}} = \frac{\exp(-\sqrt{2}\tau - 1)}{1 + \eta} \\ N_{20}^\circ &= \frac{n_{20}\exp(-\sqrt{2}\tau - 1)}{n_{10} + n_{20}} = \frac{\eta\exp(-\sqrt{2}\tau - 1)}{1 + \eta} \\ u_0 &= \tau + {}^1/_2\sqrt{2}, \qquad \eta = n_{20}/n_{10}, \qquad N_0 = n_{10} + n_{20} \end{aligned} \tag{2.2}$$

Substitute (2.1) and (2.2) in Eqs. (1.6). Assuming that δv_i and δn_i are small, and discarding all but linear terms in them and in $\varepsilon = 1 - \gamma$, the following system of linear equations is obtained:

$$\begin{gathered} (1 + \eta)\exp(\sqrt{2}\tau + 1)(\sqrt{2}\delta n_1 + d\delta n_1/d\tau) + \sqrt{2}\,d\delta v_1/d\tau - 2\delta v_1 = 0 \\ (1 + \eta)\exp(\sqrt{2}\tau + 1)(\sqrt{2}\delta n_2 + d\delta n_2/d\tau) + \eta(\sqrt{2}\,d\delta v_2/d\tau - 2\delta v_2) = 0 \quad (2.3) \\ \exp(\sqrt{2}\tau + 1)[\sqrt{2}(\delta n_1 + \delta n_2) + d(\delta n_1 + \delta n_2)/d\tau] + \sqrt{2}\,d\delta v_1/d\tau + 2\delta v_1 = 0 \\ \exp(\sqrt{2}\tau + 1)[\sqrt{2}(\delta n_1 + \delta n_2) + d(\delta n_1 + \delta n_2)/d\tau] + \sqrt{2}\,d\delta v_2/d\tau + 2\delta v_2 = \sqrt{2}(\gamma - 1) \end{gathered}$$

The solution of this system is

$$\begin{aligned} \delta v_1 &= \frac{B\eta}{\eta + 1}\exp(-\sqrt{2}\tau) + \frac{(1 - \gamma)\eta}{2\sqrt{2}(\eta + 1)} \\ \delta v_2 &= -\frac{B\eta}{\eta + 1}\exp(-\sqrt{2}\tau) - \frac{(1 - \gamma)(\eta + 2)}{2\sqrt{2}(\eta + 1)} \\ \delta n_1 &= -\frac{2\sqrt{2}\eta B}{(\eta + 1)^2}\exp(-2\sqrt{2}\tau - 1) + \frac{(1 - \gamma)\eta}{\sqrt{2}(\eta + 1)^2}\tau\exp(-\sqrt{2}\tau - 1) + C_1\exp(-\sqrt{2}\tau) \\ \delta n_2 &= \frac{2\sqrt{2}\eta B}{(\eta + 1)^2}\exp(-2\sqrt{2}\tau - 1) - \frac{(1 - \gamma)\eta(\eta + 2)}{\sqrt{2}(\eta + 1)^2}\tau\exp(-\sqrt{2}\tau - 1) + C_2\exp(-\sqrt{2}\tau)\,. \end{aligned} \tag{2.4}$$

The constants of integration B and C_i are found from the conditions on the weak discontinuity, where

$$u_1 = u_2 = 0, \qquad N_1^\circ = \frac{n_{10}}{N_0} = \frac{1}{1+\eta}, \qquad N_2^\circ = \frac{n_{20}}{N_0} = \frac{\eta}{1+\eta}.$$

It was shown above that, in the coordinate system connected with the discontinuity, its coordinate $\xi = 0$ corresponds to $x/t = -a$, where a is the velocity of the weak discontinuity in the plasma. This means that, in variables τ, the coordinate of the discontinuity is

$$\tau_0 = -\left(\frac{\gamma\eta+1}{2[1+\eta]}\right)^{1/2}.$$

Since the system of linearized equations has been solved, the expression for the velocity of the discontinuity has to be linearized with respect to ε; after doing this, we get

$$\tau_0' = -\frac{1}{2\sqrt{2}}\left(1 + \frac{\gamma\eta+1}{\eta+1}\right).$$

Substitution of solutions (2.2) and (2.4) in (2.1) gives the same value τ'_0, and also the following values of the constants:

$$B = \frac{\gamma-1}{\sqrt{2}}\exp\left[-\frac{1}{2}\left(1+\frac{\gamma\eta+1}{\eta+1}\right)\right]$$

$$C_1 = \frac{1}{1+\eta}\left\{\exp(\sqrt{2}\,\tau_0') - \frac{1}{e}\left[1 + \frac{(1-\gamma)\eta(6+7\eta-\gamma\eta)}{4(\eta+1)^2}\right]\right\} \qquad (2.5)$$

$$C_2 = \frac{\eta}{1+\eta}\left\{\exp(\sqrt{2}\,\tau_0') - \frac{1}{e}\left[1 + \frac{(1-\gamma)(\eta^2(1+\gamma)+2\eta(\gamma-2)-4)}{4(\eta+1)^2}\right]\right\}.$$

With the coefficients (2.5), the solutions (2.4) for the added terms δv_i and δn_i satisfy the required conditions; when $\gamma = 1$, $\delta v_i = 0$, $\delta n_i = 0$, as follows directly from (2.5); when $\gamma \lessapprox 1$, the added terms are small, since they are proportional to $\varepsilon = 1 - \gamma$.

Consider the behavior of the ion velocities $u_i = u_0 + \delta v_i$ for different τ, as a function of the masses of the ions and the mass ratio, i.e., as a function of γ. The weak discontinuity, representing the boundary between the undisturbed and the expanding plasma, moves with velocity a, less than the velocity in the case of a plasma with one "light" sort of ion ($a_2 < a < a_1$); and hence, the velocities of both the light and the heavy ions, departing relative to the discontinuity, are less than in the limiting case, and the difference increases as $\gamma = M_1/M_2$ falls. More detailed analysis shows that, when u_1 and u_2 are compared with $u = \tau + {}^1/_2\sqrt{2}$ at points equidistant from the relevant discontinuity, u is greater than u_2 and less than u_1 everywhere, except for a small region immediately adjacent to the discontinuity.

To sum up, the velocities u_1 and u_2 are the same on the weak discontinuity, in fact, they are zero relative to the plasma; but the accelerations $du_1/d\tau$ and $du_2/d\tau$ are different, in fact, $du_1/d\tau > 1$, $du_2/d\tau < 1$. This implies that the light ions immediately acquire a higher velocity than the heavy ions, as is amply confirmed by analysis of their behavior at the point $\tau = 0$; this latter is a singular point, since the velocity and density of each sort of ion remains constant at it at any given instant. For a plasma with one sort of ion, the values were $u^\circ = {}^1/_2\sqrt{2}$, $N^\circ = {}^1/_2 e$. Comparison of the ion velocities in the present problem with u° gives the inequality $u_1^\circ > u^\circ > u_2^\circ$, i.e., the different sorts of heavy particles travel with different velocities (though again, constant in time) at the point $\tau = 0$. As $\tau \to \infty$, the difference between the velocities of the light and heavy ions diminishes, though we always have

$$u_1 - u_2 > {}^1/_2(1-\gamma)\sqrt{2}$$

i.e., the plasma stratifies into layers of light ions moving in front and heavy ions behind. This fact is proved not only by the fact that $u_1 > u_2$ for all τ but also by the fact that $du_1/d\tau > 1$ while $0 < du_2/d\tau < 1$ at any point, though both accelerations tend to unity as $\tau \to +\infty$.

In the light of these remarks, it can be said that the momentum-wise separation that takes place almost from the very beginning of the expansion in a cold plasma with ions of one kind, in the present case only sets in at an exponential rate, and not immediately; and furthermore, it is different for the light and the heavy ions, because their masses are different.

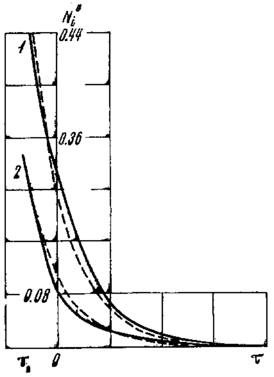

Fig. 1

This delay in τ in stabilizing the ion accelerations, which is of the same order as in the previous problem, may be explained by the electric field establishment process taking longer (in τ), because the ion distributions are more complicated than in the earlier case (due to the different ion masses). The solution has another important feature. While all the derivatives $du_i/d\tau$, $du_i/d\tau^*$ become equal in the limit as $\tau \to \infty$, the velocities themselves, or more precisely, the characteristics of Eqs. (1.4), concentrate about different asymptotes; from [1], in the case of u, the asymptote is the straight line $u = \tau + {}^1\!/_2\sqrt{2}$. In the present problem we have the straight line $u_1 = \tau + {}^1\!/_2\sqrt{2} + (1 - \gamma)\eta/2\sqrt{2}(\eta + 1)$ for the light ions, and $u_2 = \tau + {}^1\!/_2\sqrt{2} + (1 - \gamma)\ (\eta + 2)/2\sqrt{2}(\eta + 1)$ for the heavy ions.

Analysis of the solutions obtained for $N_1°$ and $N_2°$ is more difficult, since the initial concentrations n_{10} and n_{20}, i.e., the value of the parameter η, play an important role here. Three possible cases may be considered.

1. Let $\eta = 1$. At each instant the separation region can be split into two, a region of negative τ, located between the discontinuity and the boundary $x = 0$, of very small size (since $|\tau_0'| < {}^1\!/_2\sqrt{2}$), and a region $\tau > 0$, which is of much more interest. In the first region there is a very slight preponderance of heavy particles, as is confirmed by the inequalities obtained by analyzing the solution

$$\left[\frac{dN_2°}{d\tau} - \frac{dN_1°}{d\tau}\right]_{\tau=\tau_0'} \sim 0.7\varepsilon \qquad [N_2° - N_1°]_{\tau=0} \sim 0.05\varepsilon .$$

With $\tau > 0$, $N_1°$ becomes greater than $N_2°$, since the main contribution is here provided by terms of the form $\tau\exp(-\sqrt{2}\tau)$, which, for the light ions, decrease the concentration fall-off with increasing τ, and for the heavy ions, increase this fall-off.

2. When $\eta < 1$, we already have $N_1° > N_2°$ on the discontinuity, due to the greater number of light ions initially, and this inequality can only become stronger as $\tau \to +\infty$.

3. When $\eta > 1$, the heavy ions have a greater concentration in the first region than the light ions; at the point $\tau = 0$, the difference between the concentrations is greater than when $\eta = 1$. In the second region, therefore, the difference will be maintained in favor of the heavy ions for "longer" in τ (as compared with the first case), though a point is always reached, as $\tau \to +\infty$, such that $N_1°$ becomes greater than $N_2°$ regardless of the initial concentrations.

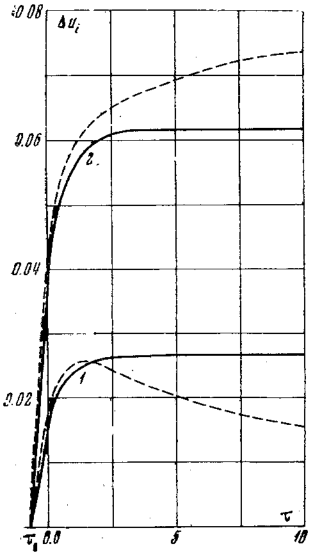

Fig. 2

The influence of the parameter γ on the functions u_i and $N_i°$ is quite straightforwad: the stronger the inequality $M_1 < M_2$ (within the context of the limiting case $\gamma \sim 1$), the greater the difference between the light and heavy ion velocities and concentrations, and the more marked become the effects described above.

On analyzing the behavior of the other dimensionless quantities characterizing the plasma expansion into a vacuum, i.e., the potential $\varphi = \ln(N_1° + N_2°)$ and the force $F = -d\varphi/d\tau$, they are found to differ somewhat from the corresponding parameters (Φ and F) in the problem with ions of one sort. Immediately behind the discontinuity, φ becomes greater than Φ. As τ increases, their difference increases, though the relative difference $(\varphi - \Phi)/\Phi$ has a maximum (~10%) at $\tau \sim 1$. As $\tau \to +\infty$, the potential and force have the same asymptotic behavior as before, i.e., $\varphi \to -\sqrt{2}\tau$ and $F \to \sqrt{2}$; the only difference is that the above-mentioned "τ delay" occurs in the present problem, due to the terms of the form $\ln\tau$ in φ and the correction $1/\tau$ to F.

*As in Russian original — Publisher.

We shall not offer a complete numerical evaluation of these parameters, and confine ourselves to mentioning the influence of the electric field on the plasma movement. Strong acceleration of part of the ions by the electric field was noted in [1], especially in a variable-temperature plasma; it can be shown that, in our present problem, the mean energies of the two sorts of ions when τ is large are

$$\varepsilon_1 = T_e\left[\tau^2 + \tau\left(\sqrt{2} + \frac{(1-\gamma)\eta}{\sqrt{2}(\eta+1)}\right)\right]$$

$$\varepsilon_2 = T_e\left[\tau^2 + \tau\left(\sqrt{2} - \frac{(1-\gamma)(\eta+2)}{\sqrt{2}(\eta+1)}\right)\right] \quad (2.6)$$

and, as might be expected, the light ions are more strongly accelerated than the heavy ions.

3. Equations (1.6) were integrated numerically by Euler's method in steps of τ, starting from the point

$$\tau_0 = -\left[\frac{\gamma\eta+1}{2(\eta+1)}\right]^{1/2}$$

where the initial values of the concentrations

$$N_{10}^\circ = 1/(1+\eta), \qquad N_{20}^\circ = \eta/(1+\eta)$$

and of the velocities $u_i^\circ = 0$ were specified. An atmospheric plasma with $\gamma = 7/8$, $\eta = 3/7$, was assumed.

Computational results are quoted below for a few values of τ.

τ	$\tau_0 = -0.69$	-0.6	0.2	1.0	1.8
N_1°	0.7000	0.5957	0.1707	0.0510	0.0154
N_2°	0.3000	0.2609	0.0754	0.0198	0.0050
u_1	0.0000	0.0972	0.9148	1.7191	2.5190
u_2	0.0000	0.0841	0.8459	1.6349	2.4307

In addition, curves are plotted in Figs. 1 and 2; the continuous curves refer to the analytic, and the broken to the numerical, solution, while the indices 1 and 2 refer to the light and heavy ions respectively.

In Fig. 1 we compare the concentrations of the two sorts of ions, obtained in the linear approximation and by the computer solution. Figure 2 offers a similar comparison of the mean velocities u_i, or more precisely, of the differences $\Delta u_i = |u_i - u^*|$, where $u^* = \tau - \tau_0$ is the modified velocity of the ions of a single kind in a simple plasma, for which we always have $u_2 < u^* < u_1$.

The approximate and numerical solutions are seen to be in good agreement immediately behind the discontinuity ($\tau_0 < \tau < 1$). As τ increases, the divergence between them becomes more marked, presumably due to the increasing inaccuracy of the linear approximation. This is confirmed by considering the behavior of the density solutions (2.4), which contain terms of the form $\tau \exp(-\sqrt{2}\tau - 1)$; as $\tau \to +\infty$, these latter provide correction terms δn_i, which are by no means small. A similar divergence is found for the velocities; here, as $\tau \to +\infty$, a constant though small deviation is found. In addition, the numerical solution reveals a maximum of the light ion acceleration $du_1/d\tau$, i.e., of $u_1 - u^*$ (Fig. 2), in the region $\tau \sim 1$, which is not revealed by the approximate solution. Meanwhile, analysis of the initial system of moment equations (1.6) indicates that the function $d^2u_1/d\tau^2$ changes sign in the interval $[\tau_0, \infty]$, whereas $d^2u_2/d\tau^2$ retains a fixed sign. In view of this, a supplementary computer evaluation was performed for different values of the parameters γ and η, analysis of which confirmed that the solutions in fact have the features just mentioned. It should be noted that the dependence on γ (within the framework of the limiting case $\gamma \sim 1$) proved to be quite weak, especially for the ion densities.

To sum up, the numerical and approximate analytic solutions obtained above indicate that the latter holds in a more restricted range; this limits its range of usefulness and suggests that further study is needed of the nonlinear properties appearing with large τ.

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